Cockrell School of Engineering

Optimal Subsidies for Carbon Capture: A Stackelberg Game Analysis

Connor Colombe, PhD Candidate
Operations Research and Industrial Engineering
The University of Texas at Austin
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Outline

- Intro and Motivation
- Model and Analysis
  - Full Information Problem
  - Model with Imperfect Information
- Case Study: Coal Fire Power Plant
- Takeaways and Future Work

What is Carbon Capture Utilization and Storage (CCUS)?

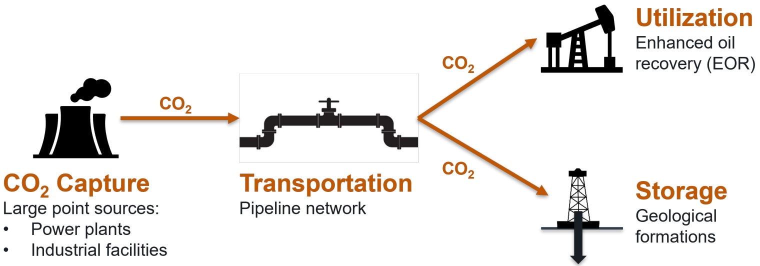
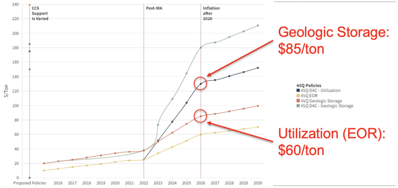
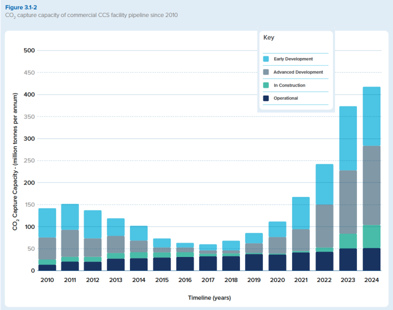


Figure: High-level depiction of CCUS Infrastructure

CCUS Policy and Deployment in the US



Progression of the 45Q subsidy



CCUS capacity growth over time

Main Research Questions

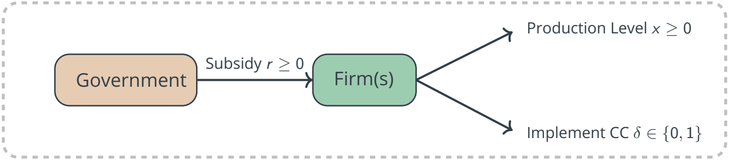
- In the context of CC subsidies, can we develop a simple model for understanding government-firm interactions?
- Given our model, can develop an expression for the social welfare maximizing subsidy and what can we learn from it?
- Could there be conditions when a CC subsidy causes a net increase in CO_2 emissions?
- How does uncertainty in CC investment costs effect subsidy values?

Literature Review

- Optimal subsidies – often analyze Stackelberg models
  - Cohen et al. (*Management Science*, 2016)
  - Chemama et al. (*Management Science*, 2019)
  - Ma et al. (*Service Science*, 2019)
  - Jung and Feng (*European Journal of OR*, 2020)
  - **Brozynski and Leibowicz (*European Journal of OR*, 2022)**

- Carbon capture – infrastructure optimization, real options
  - Middleton and Bielicki (*Energy Policy*, 2009)
  - Middleton et al. (*Env Modelling & Software*, 2020)
  - **Colombe et al. (*Energy Policy*, 2024)**
  - Fuss et al. (*Applied Energy*, 2008)
  - Yang et al. (*Energy Policy*, 2021)

Model and Analysis: Full Information Problem



- The government is the leader. It maximizes social welfare by setting the CC subsidy level r (\$/ton).
- The firm is the follower. Given the subsidy, it maximizes profit by choosing whether or not to invest in CC (δ) and how much to produce (x).
- The Stackelberg model assumes that the government can anticipate the firm's response to any subsidy level.

The Firm's Problem

$$\max_{x\geq 0} \pi(r) = \max_{x\geq 0} \left\{ \underbrace{px - f(x)}_{\text{No CC}},\ \underbrace{px + r\varepsilon x - g(x) - I}_{\text{Invests in CC}} \right\}$$

- The firm observes r, and then acts to maximize their profits based on r.
- The firm's cost functions without and with CC are $f(x)$ and $g(x)$, respectively. Assume $g(x) > f(x) \quad \forall x > 0$.
- We assume the firm is a price taker and they can sell their production at price level p.
- The CC investment cost is I and we assume $\varepsilon \in (0, 1]$ is the fraction of CO_2 captured.

The Government's Problem

$$W(r) = \max_{r \geq 0} \underbrace{\pi(r)}_{\text{Firm profit}} - \underbrace{\delta(r) \cdot r\varepsilon \cdot x(r)}_{\text{Gov spending}} - \underbrace{(1 - \delta(r) \cdot \varepsilon)\, \beta x(r)}_{CO_2 \text{ externalities}}$$

- The government's objective is to maximize the social welfare through their subsidy.
- Note that the government's problem includes the best response functions for the firm's problem $((\pi(r), x(r), \delta(r))$
- β is an externalities factor that represents the social cost of carbon.
- To find the equilibrium strategies in Stackleberg Games, we proceed via backward induction, starting with the firm's problem

Subsidy Threshold for CC Investment

Proposition 1 (*Subsidy threshold for CC investment*)

If the firm would not adopt CC at $r = 0$, then there exists some unique $\hat{r} > 0$ such that

$$\pi(r) = \begin{cases} \pi_n^*(r) & r < \hat{r} \\ \pi_c^*(r) & r \geq \hat{r} \end{cases}$$

where $\hat{r}$ is the unique solution in r of

$$p[f']^{-1}(p) - f\left([f']^{-1}(p)\right) = (p + r\varepsilon)[g']^{-1}(p + r\varepsilon) - g\left(([g']^{-1}(p + r\varepsilon)\right) - I.$$

- It's hard to gain information about $\hat{r}$ from this as is ...
- Let's introduce some simple functional forms for f and g and see what they tell us.

Specific Functional Forms

- To proceed with the analysis, we introduce some specific functional form assumptions for the cost functions:
  - Let $f(x) = \frac{a}{2}x^2$
  - Let $g(x) = f(x) + \eta\varepsilon x$
- With these assumptions, we have the following optimal production levels and subsidy threshold for CC investment:

$$x_n = \frac{p}{a} \qquad x_c = \frac{p+\varepsilon(r-\eta)}{a}$$

$$\hat{r} = \eta + \frac{p(\sqrt{2aI/p^2+1} - 1)}{\varepsilon}$$

Firm's Best Response Function

Proposition 2 (*Solution to the firm's problem*)

Under our functional form assumptions for $f(x)$ and $g(x)$, the firm's profit $\pi(r)$, production level $x(r)$, and decision to adopt CC $\delta(r)$ for a fixed subsidy level r are given by

$$\pi(r) = \begin{cases} \frac{p^2}{2a} & r < \hat{r} \\ \frac{(p+\varepsilon(r-\eta))^2}{2a} - I & r \geq \hat{r} \end{cases}$$

$$x(r) = \begin{cases} \frac{p}{a} & r < \hat{r} \\ \frac{p+\varepsilon(r-\eta)}{a} & r \geq \hat{r} \end{cases}$$

$$\delta(r) = \begin{cases} 0 & r < \hat{r} \\ 1 & r \geq \hat{r}. \end{cases}$$

Backwards Induction: The Government's Problem

Proposition 3 (*Threshold or nothing*)

Given any full information setting in which the government wishes to maximize social welfare through a CC subsidy, the optimal subsidy r^* is either: $r^* = \hat{r}$, or any $r^* \in [0, \hat{r})$.

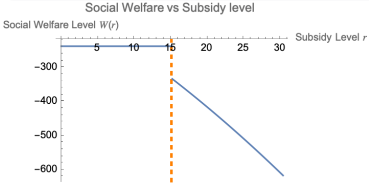


Figure: $W(r)$ when there is no investment solution $r^* = 0$.

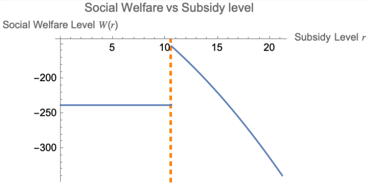


Figure: $W(r)$ when there is an investment solution $r^* = \hat{r}$.

- With full information, the government should never offer a subsidy above $\hat{r}$.

Backwards Induction: The Government's Problem

- So, the government should either set the subsidy exactly at the threshold $\hat{r}$ to induce an investment solution, or not offer a subsidy at all.
- Which one is better? We can compare $W(0)$ and $W(\hat{r})$ and take the larger one!

Proposition 4 (*Optimal subsidy level*)

For a given parameterization, if the inequality

$$p(p-\beta)+2al \leq \sqrt{2al+p^2}\,(p-(1-\varepsilon)\beta-\varepsilon\eta)$$

holds, then the subsidy level $\hat{r}$ maximizes social welfare. Otherwise, the optimal subsidy level is any $r<\hat{r}$ so that the solution will be a non-investment one.

CO_2 Emissions Impact of Carbon Capture

- Given that is optimal to incentivize CC, will net emissions actually decrease?

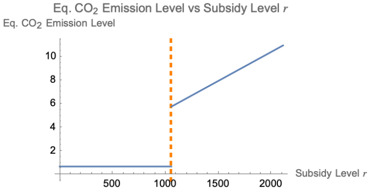
Proposition 5 (*CO_2 emissions impact of CC*)

If the government offers the threshold subsidy level $\hat{r}$, then the CC investment it induces leads to a net decrease in CO_2 emissions if and only if the following inequality holds:

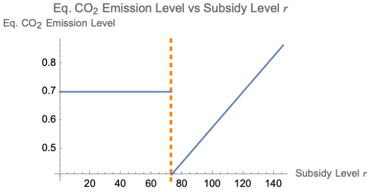
$$\varepsilon > 1 - \frac{1}{\sqrt{\frac{2Ia}{p^2} + 1}}.$$

- So, as long as the CO_2 capture fraction ε is sufficiently high, emissions will go down.
- More likely to hold when I and a are lower and when p is higher.

Illustrating Possible CO_2 Emissions Behaviors



(a) CO_2 emissions for $\varepsilon = 0.5$



(b) CO_2 emissions for $\varepsilon = 0.9$

Introducing Uncertainty

- In reality, the government does not have perfect information about the firm's CC investment cost I.
- We now consider a version of the model in which I becomes a random variable from the government's perspective, but is known to the firm.
- The government now seeks to maximize expected social welfare, $\mathbb{E}[W(r)]$.
- Now, when the government offers a subsidy r, there is some probability that is sufficient for an investment solution and some probability it is not.

Effect of Uncertainty on the Optimal Subsidy Level

- Assume that the firm's CC investment cost follows the continuous uniform distribution $\tilde{I} \sim U([I-\rho, I+\rho])$, from the government's point of view.

Proposition 8 (*Effect of uncertainty on optimal subsidy*)

If the optimal subsidy level in the full information setting is the threshold $\hat{r}$ and ΔW is the welfare difference between its investment and non-investment solutions, then in the setting with cost uncertainty from the government's point of view, the optimal subsidy level will be greater than $\hat{r}$ if

$$\rho < \hat{\rho} \equiv \frac{\Delta W \sqrt{2aI + p^2}}{(1-\epsilon)\beta + \epsilon\eta + \sqrt{2aI + p^2} - p},$$

and less than $\hat{r}$ if the inequality holds in the opposite direction.

Numerical Case Study: Coal Power Plant

- To demonstrate the application of our model, we consider a numerical case study of a coal-fired power plant and the option to retrofit it with CC.
- Assume a 10-year time horizon for decision-making.
- For all of the model parameters we can look to the literature, industry reports, and recent CC projects for reasonable values to assume.

Parameter	**Description**	**Sensitivity Range**	**Baseline**
I	Fixed CC investment cost	[0.5–2] Billion USD	1 Billion USD
η	Variable CO_2 capture cost	[50–100] USD/MWh	75 USD/MWh
β	SCC · CO_2 intensity	[50–190] USD/MWh	100 USD/MWh
p	Average price of electricity	[30–120] USD/MWh	60 USD/MWh
ε	CO_2 capture fraction	[0.75–0.95]	0.85
a	Cost function coefficient	0.4×10^{-7}	0.4×10^{-7}

Optimal Subsidy in Full Information Setting

- In the full information setting for the coal power plant case study, we find that $\hat{r}$ =\$83/ton and that this level maximizes social welfare.
  - So, the current 45Q tax credit value of \$85/ton should induce CC investment and is very close to optimal.

- Considering Proposition 5 (Threshold on CO_2 emissions), in the coal power plant case study, inducing CC investment at $\hat{r}$ decreases CO_2 emissions as long as more than approximately $\hat{\varepsilon} = 10\%$ of CO_2 is captured, which should certainly be the case in reality (our baseline value is $\varepsilon = 85\%$).

Status Quo vs. Optimal Subsidies

Metric	No Subsidy	Full Information	Cost Uncertainty
Subsidy Level (\$/ton CO_2)	0	83.4	87.8
Firm Production Level	131.4	147.1	155.2
Firm Profit (Billion \$)	3.9	3.9	4.5
CO_2 Emissions (Million Mt)	131.4	22.1	23.87
Government Expenditure (Billion \$)	0	10.4	11.6
Social Welfare (Billion \$)	-11.8	-8.1	-8.9

- Note that these results describe totals over the assumed 10-year analysis timeframe.

Key Findings and Takeaways

- In a world with perfect information, the government maximizes social welfare by offering a CC subsidy that is just high enough to induce investment, or effectively not subsidizing CC at all.
  - Our numerical case study of a coal power plant suggests that the current 45Q level of $85/ton might be very close to the optimal, threshold level.
- It is theoretically possible for a CC subsidy to increase CO_2 emissions – and we analytically established a condition under which this occurs – but this outcome seems unlikely in most real-world applications.
- When the government is uncertain about the firm's true CC investment cost, the optimal subsidy level could be higher or lower than it is in the case with full information.

Future Research Directions

- Given the drawbacks of using a 45Q-style subsidy to promote CCUS development, we could analyze other policy instruments designed for this purpose, including portfolios of multiple instruments.
- In reality there are many firms who might invest in CC in response to government incentives, so we could expand the lower-level problem to include multiple followers.
- Using the uniform distribution to describe the government's uncertainty was simple and analytically convenient, but how can we efficiently gather information to accurately represent uncertainty in this type of model?