ORIE Problem Seminar

Seminar 1: Pilot

March 6, 2023

Problem 1 (CMU Problem Solving Seminar). There's a bag with 1001 marbles, of which 501 are white and 500 are black. You can take any number of them out of the bag. If you take out the same number of black and white marbles, then you earn a number of dollars equal to the number of marbles you took out. What should you do?

Problem 2 (CMU Problem Solving Seminar). Consider a random walk in which steps of unit length are taken, where each step is in a uniformly random direction, independent of all other steps. What is the expected value of the squared distance from the origin after n steps?

Problem 3 (Project Euler). In the United Kingdom the currency is made up of pound (£) and pence (p). There are eight coins in general circulation:

$$1p, 2p, 5p, 10p, 20p, 50p, £1 \ (100p), \text{ and } £2 \ (200p).$$

It is possible to make £2 in the following way:

$$1\times£1 + 1\times50p + 2\times20p + 1\times5p + 1\times2p + 3\times1p$$

How many different ways can £2 be made using any number of coins?

Problem 4 (Applied Probability). You are given n coins. Among them are $n-1$ fair coins and a coin that has heads both sides. You randomly choose a coin, flip it k times, and observe that each flip resulted in heads. What is the probability you chose the unfair coin?

Problem 5 (CSES). You are given an array A of n integers, and an integer x. Your task is to design an efficient algorithm to find two values of A (at distinct positions) whose sum is x. Can you do this in $\mathcal{O}(n \log n)$ computations? $\mathcal{O}(n)$ computations?