Probably Probability

Problem 1) (CS388R) (a) You flip an unbiased coin until you get two heads in a row. What is the expected number of coin flips you make?

(b) You flip an unbiased coin until you get m heads in a row. What is the expected number of coin flips you make? *Challenge: What about for an arbitrary sequence of heads and tails of length m?*

Problem 2) (CS388R) You flip an unbiased coin m times. During this process, we say that the state is"balanced" if you have observed equally many heads and tails. Up to constant factors, how many times in expectation will the state be balanced?

Problem 3) (Balls and Bins, CS388R) You have n bins and n balls. You place each ball into a bin uniformly at random.

(a) What is the expected number of balls in the first bin?
(b) What is the expected number of empty bins?
(c) *(Hard)* Let X_i be the number of balls in bin $i \in [n]$. Define $Z = \max\{X_1, \ldots, X_n\}$ what is the expected value of Z?

Problem 4) (Applied Probability) Let's play a safer version of Russian roulette (with a Nerf Gun). A Nerf bullet is put into an empty 6-chamber Nerf gun and shuffled. Two players take turns pulling the trigger pointing the Nerf gun at themselves without further spinning; until the gun goes off and the player loses. If you can choose to go-first or second, what should you choose to maximize your odds of survival? And what is your probability of loss?

Problem 5) (Zhou) Jason throws two darts at a dartboard, aiming for the center. The second dart lands farther from the center than the first. If Jason throws a third dart aiming for the center, what is the probability that the third throw is farther from the center than the first? Assume Jason's skillfulness is constant. *Challenge: What if, after the first, his next $n-2$ throws are further from the center than the first? What is the probability that the n-th throw is farther from the center than the first?*

And now for some variety!

Problem 6) (Ammar) We have 16 fuel powered motorcycles all at the same starting point, each with a rider. Each of the motorcycles has a range of 100km given its current (full) fuel level. The motorcycles can easily transfer fuel to one another. If the riders work together optimally, what is the furthest distance any single one of them can get from the origin? What is their strategy? Can you generalize to n riders/motorcycles with range d km?

Hints: