Problem 1) Bob chooses an integer uniformly at random between 1 and 1000. Alice has to guess the chosen number as quickly as possible. Bob will let Alice know whether her guess is smaller than, larger than, or equal to his number. If Alice's guess is smaller than Bob's number, Bob replaces the number with another integer chosen uniformly at random from $[1, 1000]$. Prove that there exists a strategy that Alice can use to finish the game in such a way that the expected number of steps is smaller than 45

Problem 2) Assuming that temperature is a continuous function, show that at any given time on the earth's equator, there are two points directly opposite points that have the same temperature.

Problem 3) The n consecutive integers from 1 to n are written in a row. When can you place signs + and − in front of them so that the expression obtained is equal to 0?

Problem 4) I'm going to give each of you a hat to wear that is either black or white. You cannot see a hat on your head, but you can see a hat on anyone else's head. Your Goal as a group is to organize yourselves in a line, with all the white-hatted folks on one end, and all the black-hatted folks on the other end. Note that after I give you your hats, you cannot communicate with (nor touch) anyone. However, before I give you the hats you may jointly decide on a strategy. Devise a strategy that will achieve the Goal. Can you find a symmetric strategy, i.e., one that is the same for all players?

Hints:

1. Can you come up with a very simple strategy that does not work and then tweak it so that it does?
2. Use temperature and location on the equator to define a new function whose roots may be important.
3. Experiment with small values of n. Is there a pattern?