Problem 1) Let $n \geq 3$ be an integer. Alice first chooses an integer x among $1, 2, \ldots, n$. Bob knows x and chooses a different integer y among $1, 2, \ldots, n$. A fair n-sided die is then thrown which uniformly picks an integer z among $1, 2, \ldots, n$. The player whose integer is closer to z wins $\$z$. For which n is it better to be Alice than Bob?

Problem 2) You're about to get on a plane to Seattle. You want to know if it's raining. You call 3 random friends who live there and ask each other if it's raining. Each friend has a 2/3 chance of telling you the truth and a 1/3 chance of messing with you by lying. All 3 friends tell you that "Yes" it is raining. What is the probability that it's actually raining in Seattle? What about if you did this with n friends that tell the truth with probability p and they all said yes?

Problem 3) A clock's hands align at noon. When is the next time they do this?

Problem 4) 100 prisoners stand in line, one in front of the other. Each wears either a red hat or a blue hat. Every prisoner can see the hats of the people in front – but not their own hat, or the hats worn by anyone behind. Starting at the back of the line, a prison guard asks each prisoner the color of their hat. If they answer correctly, they will be pardoned. Before lining up, the prisoners confer on a strategy to help them. The prisoners, once given hats, cannot communicate, but they can hear the answers given by the other prisoners. What is their best strategy and how many people can they pardon?

Hints: