

New year new problems

Problem 1) (Crack)

- a) You have a string-like fuse that burns in exactly one minute. The fuse is inhomogeneous, and it may burn slowly at first and then quickly, then slowly and so on. You have a match and no watch. How do you measure exactly 30 seconds?
- b) Same as before, but now suppose you have two of these fuses. How do you measure exactly 45 seconds?

Problem 2)(Crack) How many consecutive zeros are at the end of $100!$ (100 factorial)? For example, $12! = 479,001,600$ has two consecutive zeros at the end.

Problem 3)(Crack) Why is it that if p is a prime number greater than 3, then $p^2 - 1$ is always divisible by 24 with no remainder?

Problem 4)(Crack) You are bidding B for an asset whose unknown true value S is uniformly distributed between 0 and 1. You know that as soon as people learn you have bid, the news will cause the value of the asset to double to $2S$. Your bid however will only be accepted if it is at least as large as the original value of the asset. How do you bid to maximize your expected payoff? (Note: your answer B may not be unique).

Problem 5)(Crack) You are seated at a table with 100 quarters on it. You are blindfolded but told that 90 quarters are tails up and 10 are heads up. While blindfolded, how do you split the quarters into two piles with the same number of heads up in each pile?
