Problem 1) Odd Streak of Heads On average, how many times do you need to flip a fair coin before you have seen a run of an odd number of heads, followed by a tail?

Problem 2) Coins in a Row On a table is a row of 50 coins of various denominations. Alice picks a coin from one of the ends and puts it in her pocket; then Bob chooses a coin from one of the (remaining ends); they continue in this manner until Bob pockets the last coin.

Prove that there exists a strategy for Alice to guarantee she has at least as much money as Bob.

Problem 3) Summing to 15 Alice and Bob alternatively choose numbers from among 1,2...,9 without replacement. The first to obtain 3 numbers that sum to 15 wins. Does Alice, who is the first to move, have a winning strategy?

Problem 4) Lockers numbered 1 to 100 stand in a row in the school gym. When the first student arrives, she opens all the lockers. The second student then goes through and recloses all the even-numbered lockers; the third student changes the state of every locker whose number is a multiple of 3. This continues until 100 students have passed through. Which lockers are open now?

Problem 5) (Putnam 2004) Basketball star Shanille O'Keal's team statistician keeps track of the number, S(N), of successful free throws she has made in her first N attempts of the season. Early in the season, S(N) was less than 80% of N, but by the end of the season, S(N) was more than 80% of N. Was there necessarily a moment in between when S(N) was exactly 80% of N?

Hints:

^{1.} Try setting up a recurrence for the expectation! Is the state with 1 head different from the state with 3 heads?

^{2.} This works when there is an even number of coins. Consider the parity of coin positions.

^{3.} No hint. Haven't tried it yet!

Try it with 10 lockers. Is there a pattern?
Draw a picture and use symmetry! What shape is equidistant from a line to a point?