

Hat's Off

Problem 1) Albert and Bilbert are about to play a game in which hats— either red or blue—will be placed on their heads. Each will be able to see the color of his partner's hat, but not the color of his own. At the blow a whistle Albert and Bilbert will simultaneously make a guess as to the color of his own hat. Incorrect guesses will not be punished. A win consists of at least one correct guess. What strategy can Albert and Bilbert agree upon to secure a win?

Problem 2) Following the same rules as question 1, one-hundred players will play the same game with hats that can be any of 100 different colors. (Not all colors need be used.) Each player can see the colors of her 99 colleagues. If a win consists of at least one correct guess and no penalty for incorrect guesses, devise a strategy that guarantees a win.

Problem 3) n people attend a hat party (a party where everyone wears a hat)! At some point in the festivities, everyone throws their hat into the air and they land in a big pile in the center of the room. The group is a bit drunk from the party, and everyone grabs a hat randomly from the pile. What is the probability that no single person grabbed their own hat out of the pile?

Problem 4)(no hats here) You are playing a game with a fair 6-sided dice. The die starts with side 1 facing up. On each turn, you may either:

1. Receive a payment equal to the value of the current face-up side of the dice
2. Re-roll the dice.

If you have n turns to play this game, what would your optimal strategy look like to maximize your winnings?

Alternatively, repeat the same game but the dice has been replaced with a uniform random number generator $U(0, 1)$. What should your strategy be now?

Hints:

1. Consider all four possibilities here and look for a rule strategy that would always work.
2. perhaps if the hats were numbered $0, 1, 2, \dots, 99$...
3. How many ways are there for exactly one person to get their hat back? Two? Three?...
4. Can you find a relation for when it is better to reroll or not?