# Hey, I'm Walk'n here!

**Problem 1)** Marathon Swimming was introduced as an Olympic event in 2008, with a 10 km race in open water. In total, the race takes about 2 hours and in 2012, the gap between the gold and silver medals was only 0.4 seconds! How could such a long race be so close? Perhaps people swim a bit faster when they are behind, and a bit slower when they are ahead.

Model this with a random walk along the integers, which starts at 0, and takes steps of +1 or -1, where the probability of stepping towards 0 is 0.6, and the probability of stepping away from 0 is 0.4. (When it is at 0, its probability of a +1 step is 0.6, and its probability of a -1 step is 0.4.) What is the probability that after 1 billion steps, the walk is farther than 10 steps from the origin? Round your answer to the nearest whole percent

**Problem 2)** Consider a random walk in the 2D plane in which steps of unit length are taken, where each step is in a uniformly random direction, independent of all other steps. Given that you start out at the origin, what is the expected value of the **squared distance** from the origin after $n$ steps?

**Problem 3)** If you play a game that is unfair (in your favor), in which you have a 51% chance of gaining \$1 on each turn and a 49% chance of losing \$1, what is the expected value of the maximum amount you are ever behind? (You are allowed to keep playing even if you are in debt because you'll eventually win your way out.)

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**And for some variety...**

**Problem (Bonus)** You are conducting surveys by phone. You call a family, and ask how many children they have. The family says that they have two children. You ask whether at least one of them is a girl who happened to be born on a Tuesday. The family says yes. Given all of this information, what is the probability that both of the children are girls? *Hint: It is not* $1/2$ *or* $2/3$

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Hints:

1. Try coding it!
2. Think about the movement of the $k$-th turn. Let the movement be denoted $X_k, Y_k$. How would you write the location on the $n$-th turn using this?
3. Use a recursive relation! Or, can you come up with a martingale for this game?