This Week's Problems

Problem (Company 1)

1. Suppose you are given a set of independent random variables (not necessarily from the same distribution) $X_1, X_2, \ldots, X_n$ with means $\mu_1, \ldots, \mu_n$ and standard deviations $\sigma_1, \ldots, \sigma_n$. Suppose we want to pick a set of weights $w_1, \ldots, w_n$ such that the convex combination of these random variables such that the random variable $Z = \sum w_i X_i$, is minimized in expectation. Formulate this as a mathematical program.

2. What is the optimal solution in this case? Why is this optimal?

3. Now suppose we want to minimize the variance of Z by choosing our weights? How would you go about choosing the weights now? Formulate the problem as a mathematical program. What class of optimization problem is this?

Problem 2) Marriage Problem Redux Suppose there are 100 candidates interviewing for a job. Your goal as the hiring manager is to select the one with the highest skill among the group. Each candidate has a unique skill value that is a real number randomly chosen from $[0, 1]$. You interview the candidates one by one and during the interview you are able to determine their skill exactly. However, once the interview ends you must hire them on the spot or reject them and they can no longer me considered for the role. Can you devise a strategy to maximize the probability of hiring the best candidate?

Problem 3) Winning an unfair game A game consists of a sequence of rounds; on each round either you or your opponent scores a point. You win with probability p, and your opponent wins with probability $1 - p$. The number of rounds n must be a positive even number. You know the value of p. To win the game, you must get *at least half* of the available points. You get to choose the number of rounds before the game starts. What do you choose if $p < 1/2$?

Problem 4) And a Cliffhanger! An ant is near the edge of a cliff. One step forward and it will tumble over the edge. The ant takes random steps forward and backward. The probability of taking a step away from the cliff is $2/3$ and the probability of taking a step towards the cliff is $1/3$. What is the probability that the ant escapes the cliff?

Hints:

1. What does convex mean? What is a nice property of expectation?
2. Consider working backwards! What should we do with one candidate left? Two? After each interview we make a decision, can you write down a relationship for the value of each decision? When do we pick either one?
3. Try to set up an equation for the probability of winning for a given n and p. Optimize baby!
4. A useful notation may be to introduce p_1 as the probability the ant escapes given 1 step from the edge. How could we use this? How does it relate to p_2 ? p_3? (maybe something recursive?)