Games!

Problem 1) Squid Game 10 contestants are arranged into a line on a bridge and in front of them lay ten left tiles and ten right tiles side by side. In order to cross the bridge, the contestants must cross 10 tiles, and at each step, the person in front must pick either the left or right tile to step on. However, for each left & right tile pair, there is exactly one sturdy tile and one faulty tile, but the contestants cannot tell them apart. The contestants cross the bridge in their assigned order with the first person picking either the left or right tile, and continuing to lead unless either a faulty tile is picked (resulting in elimination) or person one reaches the other side. If the first person is eliminated before reaching the other side, the person second in line assumes the lead, picking until he/she is eliminated (or reaches the other side), and so on. The winner of the game is the *first* person to reach the other side. Which position in line should a contestant choose if they are *playing optimally to win?*

Problem 2) Dice Game Alice rolls a fair 6-sided die with the values 1-6 on the sides. She sees that value showing up and then is allowed to decide whether or not she wants to roll again. Each re-roll costs $ 1. Whenever she decides to stop, Alice receives a payout equal to the face of the last die she rolled. Note that there is no limit on how many times Alice can re-roll. Assuming optimal play by Alice, what is her expected payout on this game?

Problem 3) There are n people in a room, each holding two pies. Each person takes aim at two other people in the room and throws their pies at them. What is the fewest number of people n such it is possible that no two people throw pies at each other? *Challenge: What if each person had k pies?*

Problem 4) Place or take You are playing a one-player game with two opaque boxes. At each turn, you can choose to either "place" or "take". "Place" places $1 from a third party into one box randomly. "Take" empties out one box randomly and that money is yours. This game consists of $N = 100$ turns where you must either place or take. Assuming optimal play, what is the expected payoff of this game? Note that you do not know how much money you have taken until the end of the game.

Hints:

1. On average, how many tiles does each person "reveal"? Can you use this to get a rough estimate for the best spot?
2. Use a strategy sets some threshold t, and optimize for t.
3. Experiment with small n, what happens?
4. Consider a strategy that only places and then only takes. What is the optimal number of takes? Why would you use a strategy like this?